Preferred Alignments of Salt Domes in Southern Louisiana

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ABSTRACT

A statistical test is described for recognizing preferred alignments in a given set of points and applied to a map of salt domes. A polygon enclosing a set of points is divided into strips. The number of points in each strip are counted as a function at the orientation of the strips. This data is used to construct the test statistic. Both artificial arrays and natural arrays, such as coastal salt domes, were tested. Salt domes in southern Louisiana have preferred alignments of N 80° W, N 80° E, and N 5° W. These alignments are respectively related to: (1) the coast line, deltaic fronts and gravity faults, (2) Lake Borgne Fault and Hancock Ridge parallel to extended Appalachian tectonic elements, and (3) the major drainage. The test is applicable to other natural data, e.g., volcanoes and epicenters of earthquakes.

INTRODUCTION

The purpose of this paper is to describe a statistical test for recognizing alignments in a given set of points. The test is applied to some artificial geometrical patterns and slightly disturbed geometrical patterns. A further application is made to the distribution of salt domes in an area of southern Louisiana (Fig. 1). The test answers the question whether or not salt domes in the given region occur along preferred lines.

STATISTICAL MODEL

Consider a region R divided into K parallel strips, each of width W. The strips are numbered from one to K as shown in Fig. 2. If σ_i is the area of the ith strip and Σ is the total area of the region R, then the probability that a point located randomly in R will fall in the ith strip is just $a_i = \sigma_i / \Sigma$.

If, on the other hand, there is a non-randomness in the selection mechanism, then the probability p_i of a point falling in the ith strip will differ from a_i for some i. In the language of statistics, our "null hypothesis" will be that the mechanism is a random one:

$$H_0 \; : \; p_i = a_i \qquad i = 1, \ldots K \tag{2.1}$$

The alternative to the null hypothesis is that for some i, $p_i \neq a_i$.

Suppose now that we have a number of points in the region R, selected by our mechanism. On the basis of their distribution we propose to either accept or reject H_0. If we do reject H_0 we will conclude that the mechanism is non-random. Among other possibilities, this may be due to a tendency for the points to align themselves parallel to the strips.

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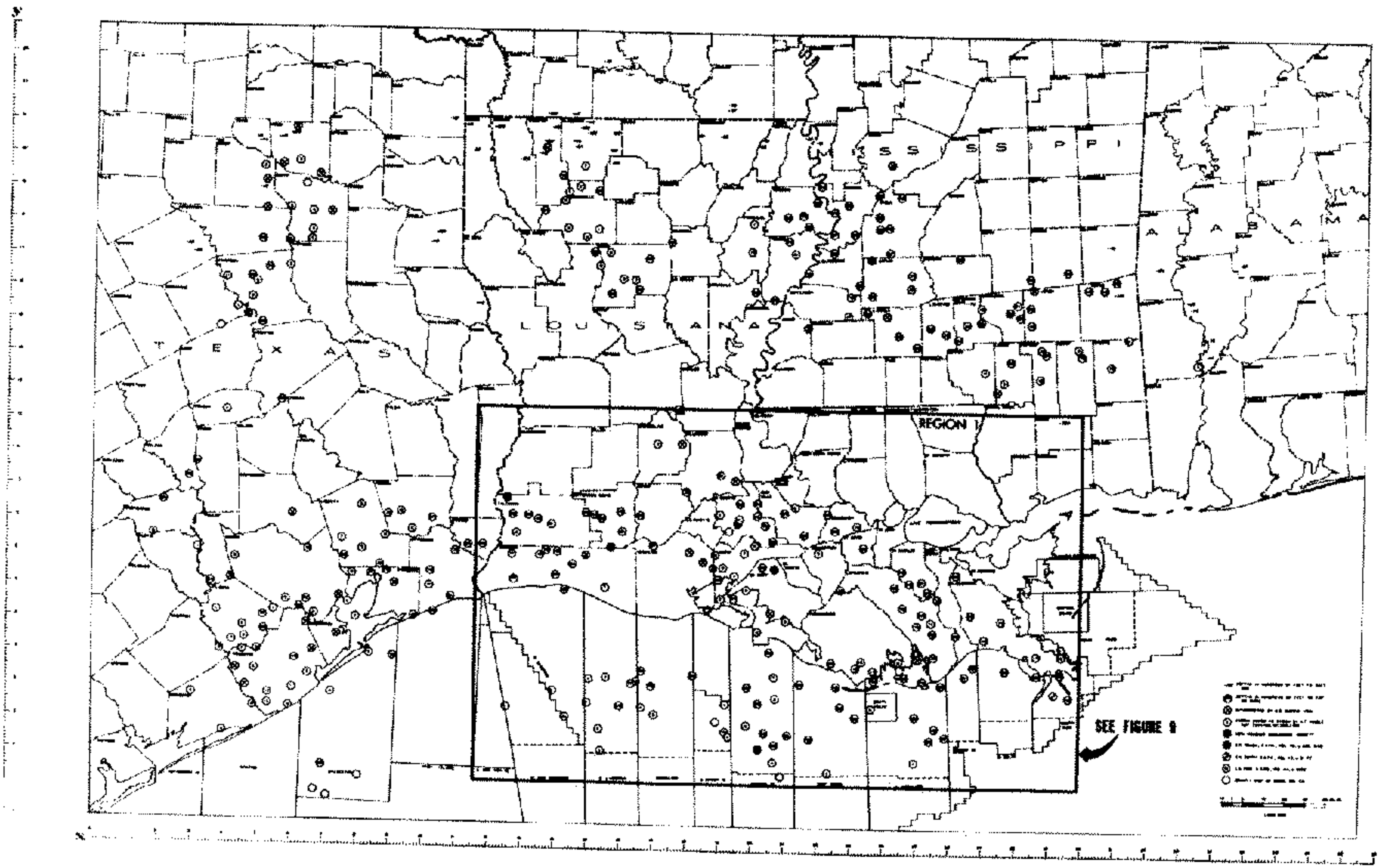

Figure 1. Location of salt domes in Gulf Coastal Province with inset showing area tested.

For testing the null hypothesis we will use the "likelihood ratio test," Mood (1950). If n_i is the number of points falling in the ith strip, then the likelihood is:

$$L(P_1, \ldots P_k) = \frac{n!}{n_1! \ldots n_k!} \prod_{i=1}^{k} p_i^{n_i} \tag{2.2}$$

where $n = \Sigma_i n_i$ is the total number of points. This likelihood is a maximum under the alternative hypothesis when $p_i = n_i/n$ and is then:

$$\frac{n!}{n_1! \ldots n_k!} \prod_{i=1}^{k} \left(\frac{n_i}{n}\right)^{n_i} \equiv L_1 \tag{2.3}$$

Under the null hypothesis the likelihood is

$$\frac{n!}{n_1! \ldots n_k!} \prod_{i=1}^{k} a_i^{n_i} \equiv L_0 \tag{2.4}$$

so that the likelihood ratio $\lambda = L_0/L_1$ is:

$$\lambda = n^n \prod_{i=1}^{k} \left(\frac{a_i}{n_i}\right)^{n_i} \tag{2.5}$$

The meaning of the likelihood ratio λ is that small values indicate that the null hypothesis is unlikely. Large values near unity mean that the null hypothesis is quite tenable. Just how small λ must be before we are willing to reject H_0 depends upon two things (Mood, 1950):

(a) The number of strips, K

(b) The tolerable "false-alarm" probability, i.e., the probability of rejecting H_0 when it is in fact true.

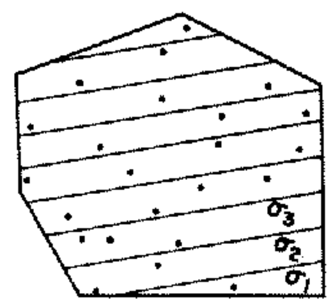

Figure 2. Test polygon.

It is a nuisance to have the rejection region depend upon K, because this in turn depends upon the orientation Θ of the sequence of strips. Values of λ for different angles are therefore not directly comparable, unless the number K of strips just happens to be the same. To circumvent this difficulty, we make one further transformation. Let $F_k(x)$ be the distribution function of λ under the null hypothesis (when n is large, $-2 \log \lambda$ has the chi-square distribution with $K - 1$ degrees of freedom). We then define the random variable U by

$$U = F_k(\lambda) \tag{2.6}$$

and note that it has the following properties:

(a) $0 \leq U \leq 1$

(b) U is a monotonically increasing function

(c) Under the null hypothesis
$\Pr\{U \geq t\} = 1 - t$

Because of (b) we can employ U just as well as λ as a test statistic. Because of (c) the rejection region is independent of K and therefore of the orientation Θ of the collection of strips. Thus values of U for different Θ are directly comparable. Finally, if the interval $[0, U_0]$ is the rejection region, then the false-alarm probability is just U_0.

To summarize, the test procedure is to first choose an acceptable false-alarm probability U_0, perhaps .05; to then compute the statistic U; and finally to reject the null hypothesis of a random distribution if $U < U_0$.

GEOMETRIC MODEL

We are given n points $P_i(x_i, y_i)$ in a plane region R. Consider a new coordinate system which is rotated through an angle Θ relative to the original axes. If (x_i, y_i) are the coordinates of P_i relative to the original axes, then (x_i', y_i') are the coordinates of P_i relative to the new axis system where

$$\begin{aligned} x_i{}' &= x_i \cos \Theta + y_i \sin \Theta \\ y_i{}' &= x_i \sin \Theta + y_i \cos \Theta. \end{aligned} \tag{3.1}$$

The family of parallel lines with equations

$$y' = c_j, \; j = 0, 1, \ldots, k-1 \tag{3.2}$$

where c_0 is chosen to make the $y' = c_0$ pass through the lowest point of the boundary of the region and $c_{j+1} = c_j + W$) will divide R into K strips. We observe that point $P_i(x_i, y_i)$ will fall into the jth strip if and only if

$$c_{j-1} < y_i{}' \leq c_j. \tag{3.3}$$

The above geometric principle will be used in applying the statistical model of Section 2 to a given sample.

APPLICATION OF THE STATISTICAL MODEL

To facilitate the application of the test to given data, a FORTRAN program was written which accomplishes the following. It divides the region into vertical strips W units wide and computes the value of the statistic U. The strips are then rotated counterclockwise through some selected increment of angle. The value of U is computed for this new position. This process is repeated until the strips have been rotated through 180 degrees. If we let Θ be the angle of inclination of the strips, then U (Θ) is the value of the statistic U for the various positions of the strips.

The graph of U (Θ) may be interpreted as follows (Figs. 3-8 and 10-12). Suppose one suspected that the salt domes in a certain region tended to occur along lines parallel to the regional fault pattern (which is known to lie along a line with an angle of inclination of Θ_0). A horizontal line is drawn on the graph through .05 on the vertical scale. In Figs. 3-10 the scale of U (Θ) is inverted so that small values of U occur at the top of the graph and large values at the bottom. If the graph is below the line at $\Theta = \Theta_0$ then no conclusion may be drawn. If the graph is above the line at $\Theta = \Theta_0$ then one is 95 per cent certain that the salt domes in the region occur with a preferred alignment along the direction $\Theta = \Theta_0$.

A second use of the graph is to provide an indication of the "amount of alignment" the given pattern exhibits along any direction from 0° to 180°. That is, the higher the peaks on the graph, the greater the alignment of the points in the pattern along these directions. However, one cannot make the probability statement of the previous paragraph about each value of Θ for which the graph exceeds the stated significance level. This is due to the fact that a pattern selected from a random process may exhibit alignments along one or more directions. See Figs. 7 and 8 for illustrations of this fact.

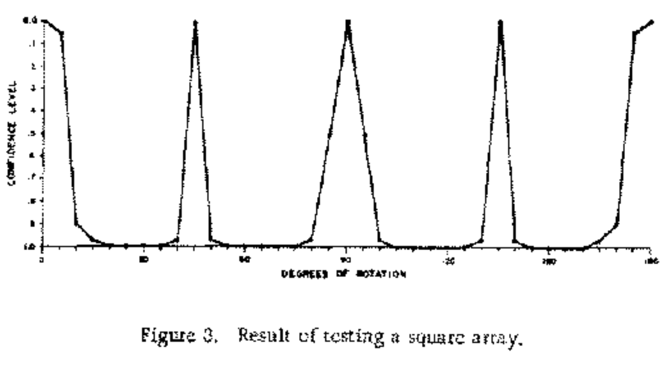

Figure 3. Result of testing a square array.

EXAMPLES

In order to evaluate the test, it was applied to five separate groups of test patterns consisting of:

1. Perfect geometric patterns (e.g., points in a square array).
2. Patterns formed by adding small random increments to the x and y coordinates of the points in the Group 1 patterns.
3. Random patterns (coordinates of the points were selected from table of random numbers).
4. Combination of 2 and 3.
5. Actual salt dome patterns from Gulf Coast Region.

In all of the following patterns, the same coordinate system is used; namely, the one used on the map of the Gulf Coastal Region (Fig. 1) on which the salt domes are plotted. However, the actual units involved should not affect the test since it is designed to measure the relative position of the points to one another. That this is in fact true, follows from the defining equation (2.6) of the statistic U. Units of measure only appear in the value p and are cancelled out since p is the ratio of units squared divided by units squared. In all the tests, the width of the parallel strips is .3125 units and the increment of rotation is five degrees.

Group 1. The first pattern in this group consists of 90 points equally spaced along a horizontal line down the center of Region 1. The values of U (Θ) were calculated as described previously. A single peak is centered about $\Theta = 0$. The value of U (Θ) is nearly 0 for those positions of the strips in which a few strips contained all the points while most of the strips contained no points. U (Θ) is nearly zero for those positions in which the points were uniformly distributed over the strips. For example, when the strips were vertical ($\Theta = 90°$).

The second pattern in this group consists of 144 points in a square array in Region 1. A square array of points has two main directions of alignment parallel to the side of the region and two minor directions of alignment along the diagonals. The results of the test on this pattern are plotted in Fig. 3. The graph has two wide peaks, at $\Theta = 0°$ and at $\Theta = 90°$, and a narrow peak at $\Theta = 45°$ and another one $\Theta = 135°$.

A third example is 146 points in a hexagonal array in Region 1. This array has three principal directions of alignment, at 0°, 60°, and 120°. It has minor directions of alignment at 30°, 90°, and 150°. The results of the test on this pattern are plotted in Fig. 4. The graph has six peaks at the above mentioned angles. However, one could not distinguish the three principal peaks from the three minor peaks from the graph alone.

Group 2. Pattern 4 (Fig. 5) consists of points (x_i, y_i) selected as follows. A set of random numbers was normalized to range from -.15625 to .15625. If we let r_i, i = 1, 2, ... be these normalized random numbers and x_i, y_i be the coordinates of the points in the square array of Pattern 2, then

$$x_i = x_i + r_i \qquad i = 1, 2, \ldots, 144$$

$$y_i = y_i + r_{i+144} \qquad i = 1, \ldots, 144$$

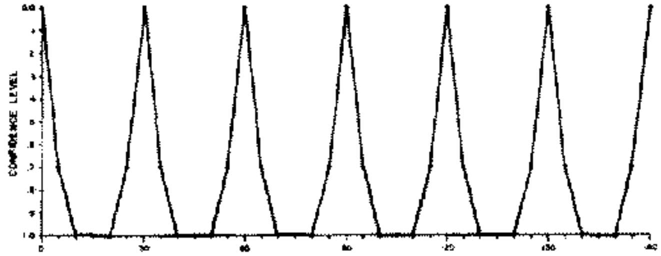

Figure 4. Result of testing a hexagonal array.

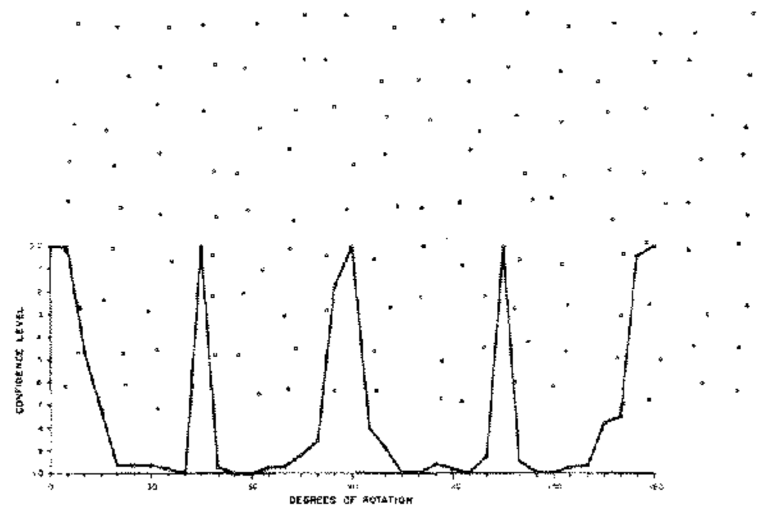

Figure 5. Result of testing a distorted square array.

The results of applying the test to Pattern 4 is plotted in Fig. 5. We point out that the width of the strips for these tests is .3125 so that the points were moved from their original position in Pattern 2 by at most, one-half of the width of the strips. This amount of distortion of the original pattern was not sufficient to greatly affect the graph (compare Fig. 5 to Fig. 3).

Pattern 5 is the same as Pattern 4, except the same random numbers were normalized to range from -0.3125 to .3125. This increased amount of distortion of the original pattern was sufficient to destroy the secondary alignment of points along the diagonals ($\Theta = 45°$ and $135°$).

Pattern 6 (Fig. 6) consists of the hexagonal Pattern 3, which has been distorted by adding random x + y increments from a set of random numbers normalized to range from -.15625 to .15625. The resulting graph is plotted in Fig. 6. This small amount of distortion was sufficient to destroy the secondary alignment along $\Theta = 30$, 90, and 150 degrees. Compare Fig. 4 with Fig. 6.

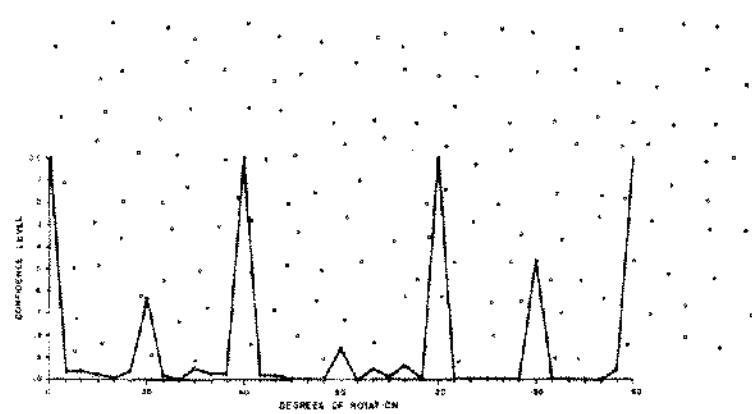

Figure 6. Result of testing a distorted hexagonal array.

Group 3. The patterns in this group were constructed as follows. A set of random numbers was normalized so that it ranged over the same interval as the x-coordinates for Region 1. Another set of random numbers was normalized to the range of the y-coordinates of Region 1. The first 146 numbers in each set were paired to form the random points in Pattern 7. The next 146 numbers in each set were paired to form Pattern 8 and similarly for Pattern 9. The graphs for Patterns 8 and 9 appear in Figs. 7 and 8 respectively.

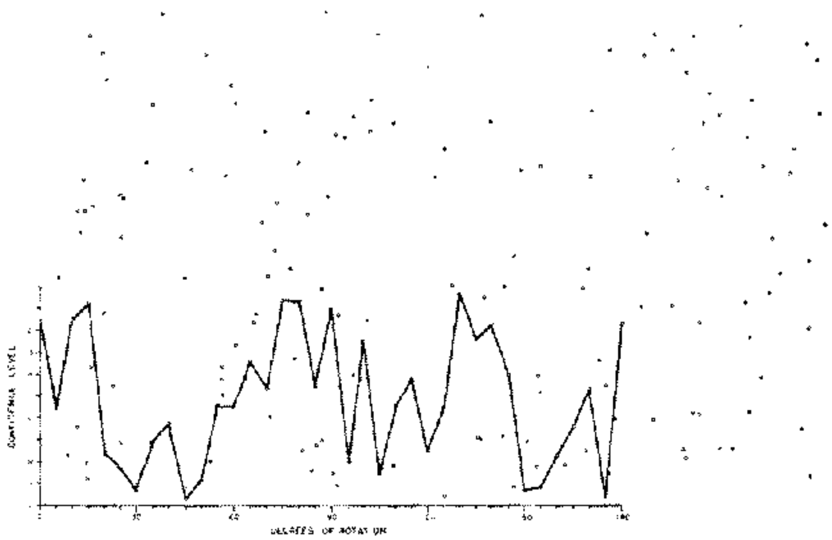

Figure 7. Random points.

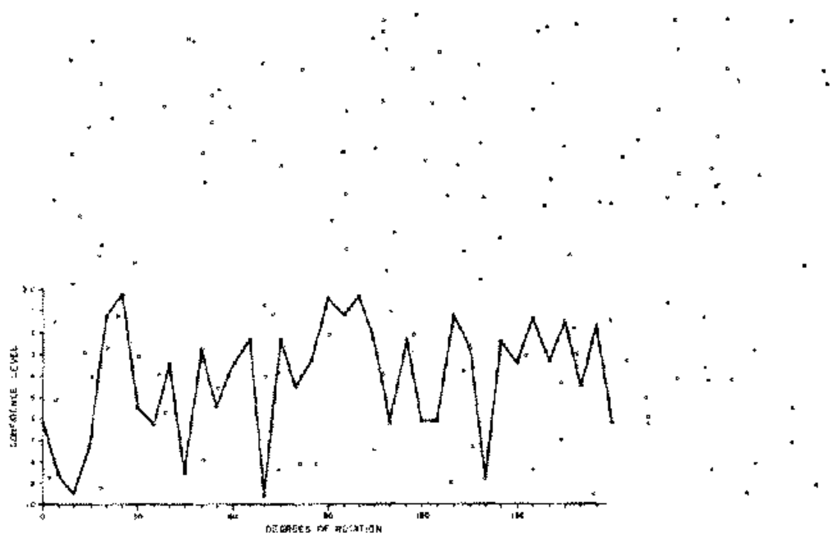

Figure 8. Random points.

Group 4. Pattern 11 consists of 144 points whose x-coordinates are the same as the x-coordinates of the points in Pattern 2 (square array) and whose y-coordinates are the y-coordinates of points in Pattern 8 (random patterns). That is, the pattern consists of nine horizontal lines with the points occurring at random on the lines. Such a pattern should have major alignment at $\Theta = 0°$ and possibly other minor alignments. In the result of this test the broad peak at $\Theta = 0°$ stands out, but the rest of the graph similar to the graph for a random pattern.

Group 5. The last step in evaluating the test was to apply it to actual salt dome patterns. Region 1 was chosen as indicated by the boundary in Fig. 1 and contains 146 salt domes. The most obvious feature of the graph of Region 1 is extremely high values from -90° to +20°. These high values are related to the choice of boundaries of the region. The northeast corner of Region 1 is void of salt domes and the southwest corner is rather sparse. Hence, when the strips are at any angle from 135° to 180° there will be many empty strips.

To remove the effect of the empty corners, a smaller region was chosen inside Region 1. The boundaries of Region 2 are shown in Fig. 9. There are 126 salt domes in Region 2. The results of the test for Region 2 are plotted in Fig. 10. There is more detail in this graph, but still relatively high values from 90° to 180°.

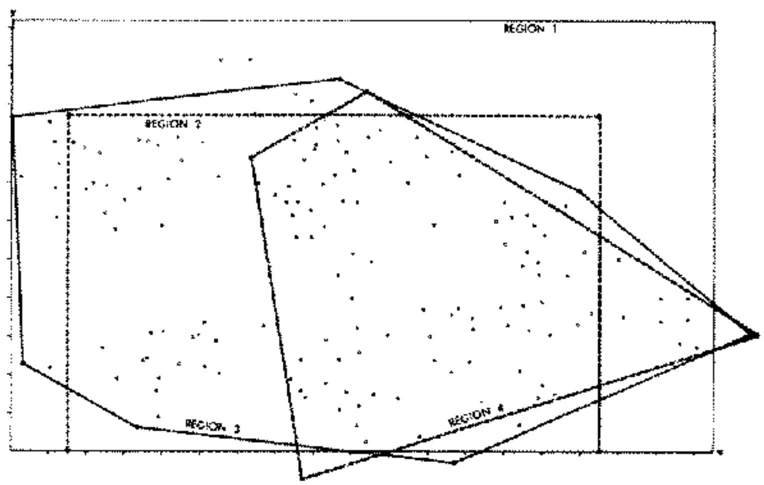

Figure 9. Regions of salt domes that were tested.

Figure 10. Salt domes in Region 2.

To further reduce the effect of empty space, Region 3 was chosen so that the boundaries contain as little empty space as possible and still keeps the region convex. This region contains all but three of the domes contained in Region 1. The resulting graph is plotted in Fig. 11.

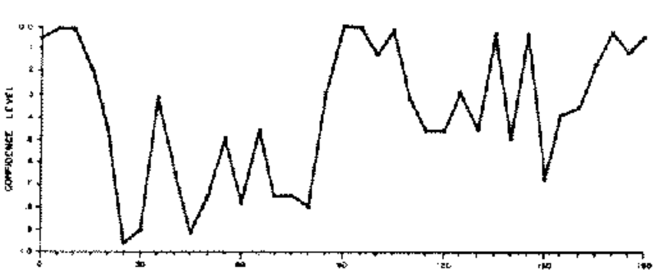

Figure 11. Salt domes in Region 3.

A still smaller region, Region 4, was chosen. It contains 93 salt domes. The resulting graph appears in Fig. 12. One observes that the principal peaks, at 100° and 10° correspond to the principal peaks for Region 3. This indicates that the salt domes over all of Region 1 have the same alignment.

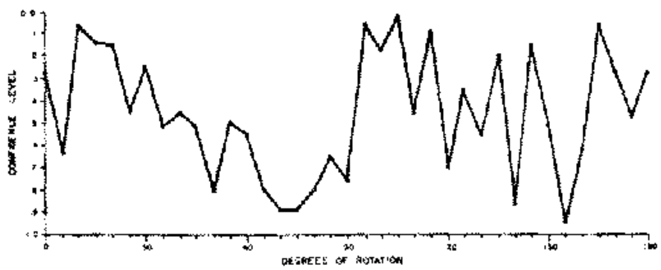

Figure 12. Salt domes in Region 4.

DISCUSSION

No obvious lineations comparable to the pure rectangular and hexagonal arrays are present in the collection of salt domes. Compare Figs. 2 and 3 with Figs. 10-12. Instead, groups of salt domes yield curves more similar to the random artificial arrays. Three possible alignments of salt domes are suggested in the test area. Older deltaic fronts and gravity faults nearly parallel the modern coast line (Murray, 1961) and there is an apparent tendency for salt domes to parallel the coast line (Selig and Wermund, in press). Using 95 per cent confidence limits, the latter is supported by peaks at $162° < \Theta < 172°$ in Regions 1 and 2, at $\Theta = 170°$ in Region 3, and at $\Theta = 165°$ in Region 4 (Figs. 10-12).

A second alignment is suggested by the test and appears at $90° < \Theta < 96°$ in Regions 1-3 and at $\Theta = 95°$ in Region 4. Neither a geologic lineament nor an alignment of salt domes having this strike (N 5° W) has been previously mentioned in the literature. This alignment is coincidental with major drainage in the region. The Mississippi River has locally stripped as much as 400 feet of sediment along its channel according to Fisk (1944). It is not known whether local density contrasts resulting from stream erosion are enough to effect the distribution of salt domes. This preferred orientation also parallels a suggested regmatic shear zone (Murray, 1961, p. 82) which might control both drainage and salt dome distribution.

A third alignment strikes approximately N 80° E. It is present at $0° < \Theta < 10°$ in Regions 1 through 3, at $\Theta = 10°$ in Region 4. This trend nearly parallels the Hancock Ridge and Lake Borgne Fault which are described by some geologists to be extensions of Appalachian structures.

It should be noted that the above alignments are significant only in the area tested, i.e., they are local elements until tested further. By traversing the entire Gulf Coastal Province with polygonal tests, it may be possible to separate regional and local alignments. Indeed, one might test any collection of points for regional and local alignments.

CONCLUSIONS

The derived test for preferred alignments is useful for examining geologic data points where the geologic setting is known. Because a suggestion of alignment can appear in random distributions, indications of geologic alignment must be carefully examined within a total geologic framework.

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